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Asset Allocation within Variable Annuities: The Impact of Guarantees

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Asset Allocation within Variable Annuities: The Impact of Guarantees

Abstract

The latest generation of variable annuity contracts contains equity put options plus longevity insurance. The marketing material for these products often claims that these new riders should induce purchasers to take on more financial risk. This chapter examines whether this is indeed the case. Using a unique database, we document that policyholders do in fact adopt higher equity exposures when these riders are selected. We also examine the theoretical merits of the marketing advice, by deriving the optimal asset allocation in the presence of these guarantees. We conclude that more aggressive equity allocations can indeed be justified in many, although not all, product structures.

Keywords

variable annuities, longevity risk, equity

Disciplines

Economics

Comments

The published version of this Working Paper may be found in the 2008 publication: *Recalibrating Retirement Spending and Saving*.

Recalibrating Retirement Spending and Saving

EDITED BY

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Chapter 12

Asset Allocation within Variable Annuities: The Impact of Guarantees

Moshe A. Milevsky and Vladyslav Kyrychenko

Variable annuities (VAs) are close cousins of mutual funds, which bundle individual securities such as stocks and bonds into diversified units or trusts. Nevertheless, they are formally classified as insurance policies, since they are sold by insurance companies and contain insurance guarantees, in addition to being registered as securities. The most recent generation of variable annuity contracts has been financially engineered to provide a range of income guarantees meant to protect the policyholder against what the industry has coined the ‘sequence of returns’ risk. This refers to the chance that a retirement portfolio from which cash is being withdrawn suffers early losses. The common denominator of all these insurance riders is that they contain an implicit put option on financial markets plus some form of longevity insurance, akin to a pure life annuity. Of course, using the concept of put-call parity, they can also be viewed as call options to annuitize at a variable strike price. It is estimated that ~70 to 80 percent of VAs currently sold contain these living benefit riders, for a total of around \$100 billion.

The promotional material for such products often claims that these guarantees should induce purchasers to take on more financial risk than they normally would without these guarantees. In fact, some of these products are referred to as a ‘bond substitute’ within a diversified portfolio, or even as a risk-free instrument. This chapter explores these new products using a unique database of policyholder behavior supplied by the Life Insurance Marketing Research Association (LIMRA). We show that VA policyholders are indeed adopting more aggressive allocations (i.e., higher equity exposures) when these riders are actually selected. We also examine the theoretical merits of this advice by deriving the optimal asset allocation—under a stylized model of these products—in the presence of these optional riders.¹

In what follows, we review some of the relevant academic literature on the topic of portfolio choice over the life cycle. Next, we describe our data and provide summary results. The subsequent section provides an analytic

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model of portfolio choice in the presence of these guarantees, and a section offers conclusions and additional observations.

Background

As of early 2007, over US \$1.2 trillion has been invested in VAs in the USA, with gross annual sales in the hundred billion dollar range; clearly it is a substantial market. VAs have long provided tax-sheltered growth and deferral; currently they also embed a number of put-like derivatives that provide guarantees on the account value. Like all insurance riders, and in contrast to standard exchange traded options, insurance companies charge for this downside protection by deducting an ongoing fraction of assets as opposed to an up-front fee. These unique features differentiate the pricing of this derivative security from the standard Black-Scholes approach where the option premiums are paid up front and in advance. (This will be important later when we examine optimal portfolio allocations.)

In what follows, we focus on a type of rider called the Guaranteed Minimum Income Benefit (GMIB). The mechanics of this option are explained analytically below, but for now it is important to know that the essence of a GMIB consists of a market put option that allows the holder of the VA to annuitize the account at a guaranteed rate—which then provides a guaranteed level of lifetime income.² At the point of purchase, the investor may select the GMIB; this rider gives the holder the ability to annuitize some minimally guaranteed amount at some contractually guaranteed rate. Thus, for example, if a \$10,000 premium were placed into a variable annuity, the insurance company might guarantee that at least \$15,000 worth of life-annuity income can be purchased in 10 years. The purchase price (or annuity factor) would be specified within the contract, for example \$20 per dollar of lifetime income. Thus this contract would guarantee a life annuity of $\$15,000/\$20 = \$750$ per year in the worst case scenario; if the market value of the (subaccounts within the) variable annuity were worth more than \$15,000 in 10 years, the policyholder could annuitize the market value at *market* annuity rates.

A number of recent papers have extended the set of decisions included in the portfolio choice problem to highlight the interaction and the risks faced by the household, broadly defined.³ The common denominator in these studies, and an oft-debated question, is how portfolio allocations to risky assets (stocks) should evolve with investors' age. Finance practitioners often recommend lowering the equity content of one's portfolio with increasing age. Much of this is at the heart of the life-cycle funds recently promoted by many mutual fund companies. A popular rule of thumb is to have the share of equities equal to 100 minus the investor's age.

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By contrast, academic analysts have built on Merton's classical result (1969, 1971) that age should *not* matter in the absence of human capital considerations. Several authors who include human capital in their models by and large support the recommendation to decrease equity share in the portfolio with increasing age, although the generated profiles are not necessarily monotonic.⁴ Other stylized facts have also emerged regarding the determinants of the optimal equity share in investors' portfolios with age. For instance, most document the importance of a declining age-equity share profile.⁵ Others document a hump-shaped pattern, with a declining part starting at the ages of 50–60.⁶

Empirical Methods

In what follows, we use data containing VA policy purchase information on 812,367 individual variable annuity contracts collected by LIMRA. That organization gathered contract and product information from 10 member life-insurance firms which sold variable annuity policies during the period January 2000–June 2004; these policies had to be in force as of June 2004 and had to *offer* at least one guaranteed living benefit (GLB) rider at the time of purchase. This data-set has several advantages compared to commonly used financial micro-level survey information such as the Survey of Consumer Finances (SCF) and the Panel Study of Income Dynamics (PSID). First, LIMRA's data-set has a definite size advantage, as it provides information on more than 812,000 annuity contracts. In comparison, PSID follows only about 8,000 families while the SCF contacts some 4,000 households in its triennial surveys. Second, LIMRA's information comes directly from insurance firms' original contracts, so they are much more accurate than self-reported information from household surveys. Third, the LIMRA data provide much greater detail with respect to asset allocation choices (e.g., amounts invested in small-cap, medium-cap, large-cap, or international stock funds, investment grade or high-yield bond funds, and balanced funds). Finally, and most importantly, the LIMRA data are unique in providing information for asset allocation within variable annuities with and without GLB riders.

Of course, as Campbell (2006) points out, there is no perfect source of household financial data. To this end, the LIMRA data do not include comprehensive information about investors' personal characteristics and other investment accounts. What the LIMRA files do provide is some 60 variables about investor and contract characteristics including the investor's age, sex, and state of residence. We also have data on the investor's account value and how it is invested, the distribution intermediary channel, whether a GLB rider was selected, and the features of each type of GLB rider. Records

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were eliminated with missing age information and those records for which the values of different sub-accounts did not add up to the account value; this reduced our sample from 812,367 to 679,579 observations. We further limit our sample to those investors who (a) either selected no GLB rider or (b) selected the GMIB only. We do this for two reasons. First, the GMIB is the closest equivalent to a life annuity with longevity insurance [compared to other GLB riders, such as the Guaranteed Minimum Withdrawal Benefit (GMWB) or Guaranteed Minimum Accumulation Benefit (GMAB)]. Second, the GMIB is by far the most popular GLB rider selected by individuals. Of all contacts we examined, where at least one GLB rider was selected, only a GMIB was selected 95 percent of the time. These exclusions produce a 'clean' data-set of 660,336 observations with either GMIB or no GLB riders selected.

Empirical Findings

Next, we describe the relationship between age and asset allocation for investors who selected GMIBs, compared to those who did not select any GLB. More specifically, we consider the percentage of the investor's VA account held in high and medium risk (HMR) assets, which we describe below as 'risky assets,' as a function of the investor's age. The following funds are included in the HMR category: large-cap, mid-cap, and small-cap stock funds, high-yield bond fund, balanced fund, specialty/sector fund, and international equity fund. Conversely low risk (LR) investments are those held in investment-grade bond funds, money-market funds, and fixed funds.

Next, we analyze the two companies referred to here as companies A and B with the highest number of annuity policies in our sample, 170,462 and 126,118 annuity contracts, respectively. These two companies combined represent 52 percent of VAs with selected GMIBs in the LIMRA sample. Figure 12-1 and Table 12-1 describe the percentages invested in HRM assets by age groups. In both cases, we present two age versus HMR results: one for investors who selected GMIB and another for those who selected no GLB rider. For company A, for example, investors who did not select any GLB rider have a declining percentage invested in HMR assets by age. The decline is almost monotonic, from 77 percent for the under-age 40 group, to 54 percent for the over-age 80 group. A very different profile emerges for investors who selected GMIB: here, the percentage invested in HMR assets declines with age much more slowly to age 71–75, and then it starts to rise. The GMIB group's exposure to HMR assets is much higher, being in the 80–86 percent range. We conclude that additional HMR (risk) exposure in the group of GMIB selectors in company A is between 11 and 31 percent.

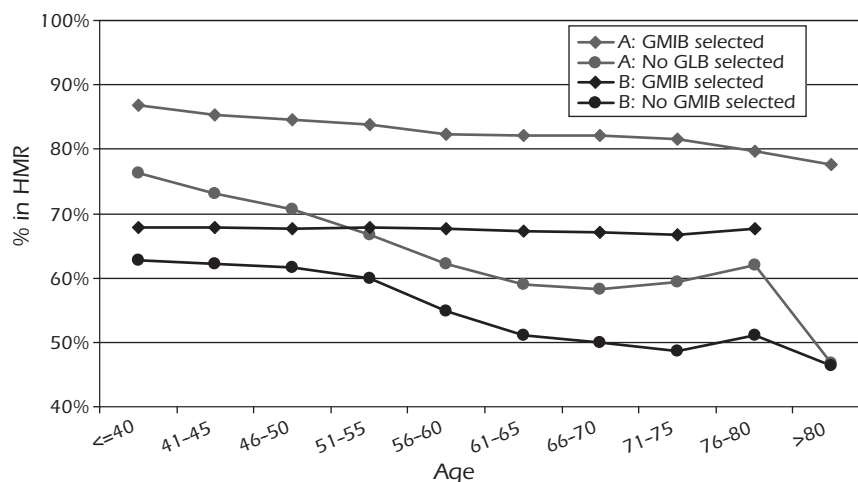
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Figure 12-1. Allocation to risky assets, with and without GMIB option. *Source:* Authors' computations. *Note:* This figure displays the percent of the variable annuity sub-account within the policy sold by Companies A and B allocated to high and medium risk (HMR) asset classes, when the GMIB is selected and when no Guaranteed Living Benefit (GLB RIDER) is selected.

The impact of age on the HMR allocation graph is similar for company B. The allocation to risky assets for investors who selected a GMIB is substantially higher and again declines with age much more slowly than for investors not selecting a GLB rider. Interestingly, the total share of assets invested in HMR assets is significantly lower for company B than for A at all age groups. Furthermore, in company B, for investors who selected GMIB, this share is very stable regardless of age. These differences are probably attributable to the different investment choices available in each company, as well as to different asset allocation restrictions applied when a GMIB rider is selected. Additional HMR exposure in the group of GMIB selectors in company B is between 5 and 18 percent.

Finally, Figure 12-2 illustrates how the selection of HMR versus LR assets varies with the distribution channel. What we mean by distribution channel is the type of intermediary selling the variable annuity product to the customer. The patterns show stark differences in the HMR equity exposure, both with and without the selection of the GMIB, depending on whether the VA was purchased through a bank, an independent financial adviser, a wirehouse broker, or some other source. For example, in company A, policies with the highest allocation to risky assets, with or without a GMIB, were sold by financial planners. Policies sold through banks have the biggest

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TABLE 12-1 Fraction of Variable Annuity Portfolio Held in High or Medium Risk (HMR) Assets

<i>Investor Age</i>	<i>Company A</i>			<i>Company B</i>		
	<i>% in HMR</i>		<i>Additional HMR Exposure (%)</i>	<i>% in HMR</i>		<i>Additional HMR Exposure (%)</i>
	<i>GMIB only</i>	<i>No GLB RIDER</i>		<i>GMIB only</i>	<i>No GLB RIDER</i>	
≤40	86.90	76.30	10.60	67.80	62.80	5.00
41–45	85.30	73.10	12.20	67.80	62.20	5.60
46–50	84.60	70.60	14.00	67.60	61.70	5.90
51–55	83.80	66.70	17.10	67.90	59.90	8.00
56–60	82.40	62.20	20.30	67.60	54.90	12.70
61–65	82.00	59.00	23.00	67.30	51.10	16.10
66–70	82.10	58.30	23.80	67.10	50.00	17.20
71–75	81.60	59.30	22.20	66.80	48.70	18.10
76–80	79.70	61.90	17.80	67.60	51.10	16.40
>80	77.50	46.70	30.80	N/A	46.40	N/A
Total no. of policies	104,377	66,085		89,949	36,169	

Source: Authors' computations, see text.

Notes: This table presents the percentage invested in high and medium risk (HMR) assets for Companies A and B, for 10 age groups in five year groupings. The first column for each company represents the percentage allocation to HMR by investors who selected a Guaranteed Minimum Income Benefit (GMIB); the second column represents allocation to HMR by those who did not select any guaranteed living benefit (GLB rider). The table shows that VA policyholders hold more HMR assets when they select a GMIB. For company A, this ranges from 10–30%; for company B this ranges from 5–16%. Note that in the case of company B, contracts were not issued above the age of 80. The HMR allocations are not directly comparable across companies due to different investment options.

difference between the percentages allocated to HMR for investors who selected GMIB compared to those who did not. In company B, the VAs with the highest HMR allocations (where no GLB rider was selected) were sold by stockbrokers, while the policies which included GMIBs were sold by financial planners. Here the largest gap in allocations to risky assets was in the policies sold by stockbrokers. Of course, this is not to say that the any of the channels necessarily influence or cause the particular allocation differences, as this could arise from differences in the clientele. People who are more 'conservative' and hence likely to hold less HMR assets might be more likely to purchase the VA through a bank, and vice versa. In our view, the most likely explanation for the asset allocation distribution channel effect is likely to be the type of customer who uses these intermediaries, as opposed to the intermediaries themselves.

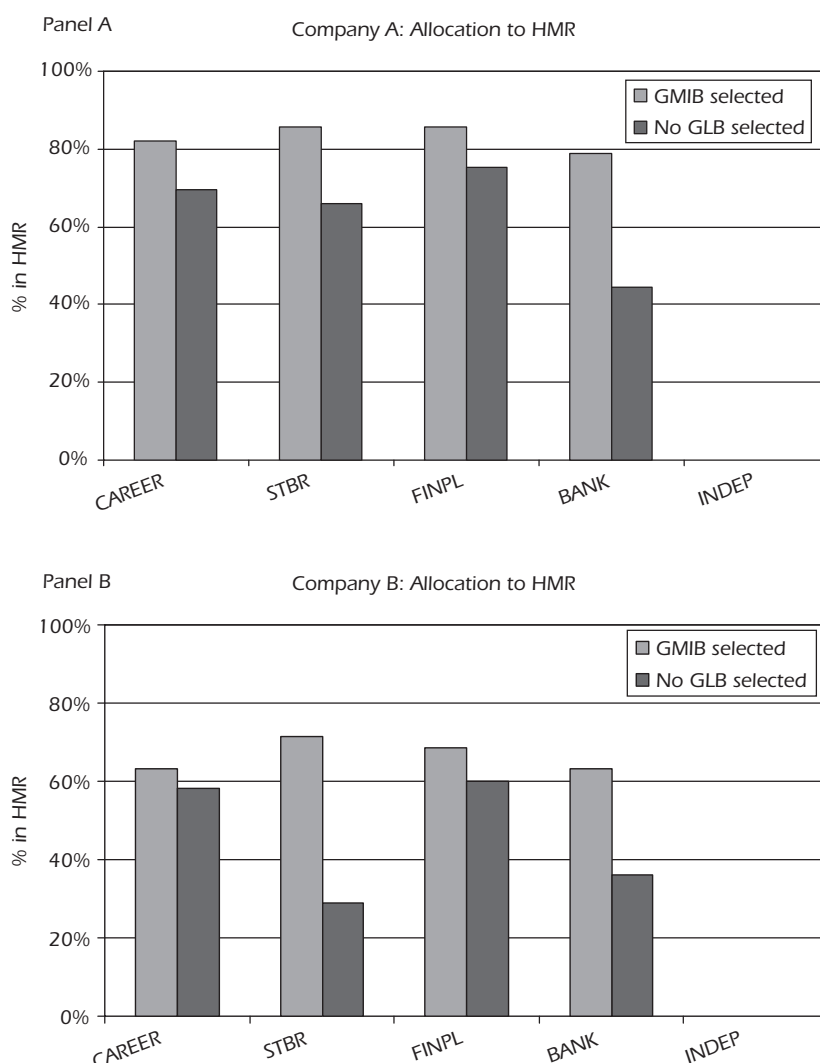


Figure 12-2. How distribution channel influences risky allocation. *Source:* Authors' computations. *Notes:* Funds regarded as high or medium risk include large-cap, mid-cap, and small-cap stock funds, high yield bond fund, balanced fund, specialty/sector fund, international equity fund. The rest of the fund classes available (including investment-grade bond fund, money-market fund, and fixed fund) are considered as low-risk assets. The HMR allocations are not directly comparable across companies due to different investment options. Variable annuities manufactured by Companies A and B can be purchased via several distribution channels including career insurance agents (CAREER), stockbrokers (STBR, including wirehouses), financial planners (FINPL), and banks. The two panels illustrate the impact of this distribution channel on the allocation to HMR assets, both with and without guarantees. Panel A: Company A's allocation to high and medium risk (HMR) assets. Panel B: Company B's allocation to high and medium risk (HMR) assets.

12 / Asset Allocation within Variable Annuities 283**Multivariate Statistical Analysis**

Next, we evaluate whether the relationship between age and HMR assets holdings observed in our previous tabulations is robust to controls for other variables. Accordingly, we employ a linear regression model where the dependent variable is the percentage invested in HMR assets. Control variables include the investor's age, as well as several others including age squared (AGE^2), to capture possible nonlinearities. We also control on the log of the investor's account value ($LOGACC$) as a proxy for investor's wealth; prior studies have reported that wealthier investors have more aggressive asset allocations. The investor's sex ($MALE$) is included as previous research has indicated that men tend to be more overconfident than women and so may invest proportionally more in risky assets. A qualitative variable (IRA) captures the tax status of the account. Since investments inside IRAs are tax sheltered, we might expect to see a higher share of fixed income assets in those accounts because interest income is taxed at a higher rate than dividends or capital gains on stocks. Of course, variable annuities are already tax-sheltered investments, so theoretically it should not make a difference in terms of the tax implications. Following our earlier discussion, we include a control for the distribution channel, or the intermediary by which the variable annuity contract was purchased. Four qualitative variables ($CAREER$, $STBR$, $FINPL$, and $BANK$) indicate, respectively, that the sale was made via an insurance agent, stockbroker, financial planner, or bank employee (the reference category is independent agent). These are included to allow for the possible impact and influence of the intermediary on the percentage of risky assets selected. Finally, we allow for company-specific controls by including seven company indicator variables for those firms that sold VA policies with GMIB riders (the omitted category is the three companies with the lowest number of VA contracts with GMIB.)

Linear regression results are presented for two subsamples: one includes only variable annuity contracts without any GLB rider (Model 1), and the second includes only those contracts where variable annuity holders selected a particular type of GLB rider, namely, the GMIB (Model 2). Table 12-2 summarizes findings. Most importantly, the negative coefficient on AGE indicates that the share invested in risky assets declines with age for both subsamples. Nevertheless, it declines much faster for those investors who did not select any GLB rider, as the AGE coefficient in Model 1 is three times larger (in absolute value) than in Model 2; their difference is highly significant based on t -test.⁷ Moreover, AGE^2 coefficient is negative in Model 1 but positive in Model 2. This suggests that the share invested in risky assets is a concave function of age which declines faster at older ages for investors who do not select any GLB rider. For those who select GMIB

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TABLE 12-2 Factors Associated with the Fraction of the VA Account Held in High or Medium Risk (HMR) Assets: Multivariate Regression Results

<i>Variable</i>	<i>With GMIB</i>		<i>Without GLB RIDER</i>	
	<i>Estimate</i>	<i>t-value</i>	<i>Estimate</i>	<i>t-value</i>
Intercept	0.913*	59.22	0.992*	131.92
LOGACC	-0.002*	-5.66	-0.018*	-30.42
MALE	0.012*	16.45	0.026*	18.24
AGE	-0.002*	-43.09	-0.004*	-76.1
AGE ²	1.8E-06*	5.57	-3.5E-06*	-8.02
IRA	-0.004*	-5.59	-0.010*	-6.37
CAREER	-0.039*	-8.8	0.076*	18.82
STBR	0.011*	2.6	0.0230*	7.27
FINPL	0.003	0.57	0.147*	31.75
BANK	-0.058*	-13.02	-0.143*	-34.42
C1	-0.133*	-9.26	-0.093*	-23.29
C2	0.038*	2.67	0.081*	21.68
C3	0.088*	6.14	0.201*	38.11
C7	0.029*	1.98	0.165*	44.29
C8	-0.049*	-3.33	-0.045*	-9.91
C9	-0.073*	-5.03	0.232*	48.5
C10	0.106*	7.38	0.206*	47.94
Adjusted R^2	0.154		0.159	
No. of observations	368,005		272,564	

Source: Authors' calculations. See text for variable definitions.

Notes: OLS regression estimates provided, where the dependent variable is the percentage invest in the HMR portfolio (see Table 12-1). The first regression is based on the subsample of variable annuity policies where no guaranteed living benefits (GLB rider) was selected; the sample pools data from seven firms. The second regression uses the subsample where longevity-put was selected; the pooled sample includes 10 companies (see text). We control for company differences using dummy variables representing the seven companies where many selected GMIBs. The omitted category is the other three companies.

* Significant at the 5% level.

products, however, the age decline is attenuated, and it even reverses at older ages. These results are generally in line with Figure 12-1.

Besides age, several other variables prove to be significant in explaining the percentage of an account invested in HMR assets in both models. We find that men do invest their VAs more aggressively than women, by holding more equity. VAs, which are part of a tax-sheltered (IRA) plan, have a lower share of risky assets and a higher proportion of low-risk bonds. This would be an intuitively pleasing result—that is, bonds are more likely to be

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placed in tax shelters—were it not for the fact that variable annuities are already tax sheltered. An unexpected finding is that account size (wealth level) is negatively associated with an allocation to HMR (risky) assets. The distribution channels variables lead one to conclude that, for investors who do not select a GLB rider, riskier portfolios result from having financial planners provide the product. For those who did select the GMIB, however, stockbrokers are those associated with relatively riskier asset allocations. In results not reported here, we also show that the company controls are statistically significant.

To explore further some of these empirical patterns, we next repeat the regression analysis for the two companies having the largest number of variable annuity policies. Results appear in Table 12-3, where we see that the age/equity patterns are similar to those in Table 12-2. That is, the share of risky assets declines with age much faster for investors who did not select a Guaranteed Living Benefit. Conversely, for investors who selected GMIB, the negative age effect is attenuated. Male investors generally allocate more to risky assets. In terms of account market value, however, the results are different depending on the company. In company A, the market value of the account is positively related to the share of risky assets, whereas in company B, the relationship is negative (similar to the result for the whole sample). Interestingly—and in contrast to the pooled results—in both companies the share invested in HMR is higher when the variable annuity is within tax-deferred account, but only when the GMIB is not selected. But when the GMIB is selected, the IRA status is associated with ‘more bonds’ and less HMR asset classes.

A Model of Portfolio Choice with a GMIB

Next, we construct a stylized model of portfolio choice in the presence of a GMIB option, so as to derive a measure for the amount of ‘extra risk’ a rational (utility-maximizing) investor would be willing to take when granted a GMIB option that protected him from downside risk. We do so by postulating a generic investor with W_0 of initial wealth initially optimally allocated α^* to risky assets, and $1 - \alpha^*$ to safe assets. At the outset, we assume that these allocations take place within a VA but one which lacks extra riders. The risky and safe assets correspond to the HMR and LR funds described in the earlier section. By selecting this particular allocation, assumed to be optimal, the investor has revealed his explicit risk preferences. We further assume constant relative risk aversion (CRRA) and lognormal asset returns, as per Merton (1969), the implied coefficient of relative risk-aversion γ will be equal to $(\mu - r)/\sigma^2$, where μ is the expected return, σ is the volatility and r is the risk-free rate of return. For example, an investor with a preexisting

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TABLE 12-3 Factors Associated with the Fraction of the VA Account Held in High or Medium Risky (HMR) Assets: Multivariate Regression Results for Companies A and B Separately

Variable	Company A				Company B			
	With GMIB		Without GLB RIDER		With GMIB		Without GLB RIDER	
	Estimate	t-value	Estimate	t-value	Estimate	t-value	Estimate	t-value
Intercept	0.639*	83.56	0.514*	28.07	0.948*	111.09	0.985*	60.72
LOGACC	0.003*	5.12	0.005*	3.71	-0.007*	-10.24	-0.022*	-16.46
MALE	0.004*	2.76	0.004	1.15	0.020*	13.28	0.036*	12.18
AGE	-0.001*	-11.21	-0.003*	-20.62	-0.002*	-24.03	-0.005*	-37.13
AGE ²	1.7E-06*	2.79	-9.1E-06*	-8.7	3.8E-06*	5.69	-9.3E-06*	-10.1
IRA	-0.005*	-3.14	0.008*	2.1	-0.005*	-3.09	0.014*	4.14
CAREER	-0.002	-0.7	0.178*	22.99	0.027*	12.09	0.217*	56.23
STBR	0.078*	22.29	-0.059*	-6.94	0.069*	27.83	0.239*	58.29
FINPL	0.051*	18.67	0.236*	31.98	0.069*	27.84	0.310*	63.13
Adjusted R ²	0.017		0.121		0.018		0.1178	
No. of observations	89,947		36,165		103,348		65,855	

Source: Authors' computations. See text for variable definitions.

Notes: OLS regression estimates provided, where the dependent variable is the percentage invest in the HMR portfolio (see Table 12-1). The first regression for each company is based on the subsample of variable annuity policies where no guaranteed living benefits (GLB) were selected. The second regression is based on the subsample from the same company where longevity put was selected. While some coefficient signs vary depending on the specific company studied, note that the age variable is always negative (with and without the GMIB) and the Male variable is positive (with and without the GMIB).

* Significant at the 5% level.

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allocation of 50 percent HMR and 50 percent LR reveals a risk aversion of exactly four, when the expected return from the HMR investments is 9 percent, the volatility is 15 percent and the risk-free (LR) rate is 4.5 percent.

We then take this so-called $\alpha^* \Rightarrow \gamma$ investor and evaluate how he might change his allocation to the HMR asset class, if he were to hypothetically be granted the annuity put option underlying the GMIB. More specifically, let α^{**} denote the new (presumably) optimal allocation to the HMR asset class when the GMIB is 'wrapped' around the investment account. Recall that the α^* is the original (optimal) allocation in the absence of this put option guarantee. The difference between α^{**} and α^* , which we define as ε , is the incremental allocation that is theoretically justifiable, based on the presence of the GMIB. We seek to investigate the behavior of this ε quantity as a function of the various underlying contractual parameters, such as the strike price of the embedded option, the preexisting allocation α^* , and other exogenous capital market parameter assumptions.

Recall the GMIB guarantees the ability to convert or annuitize (in the worst case scenario) the guaranteed amount $W_0 e^{\eta T}$ at a prespecified rate denoted by g_x , where η is a guaranteed investment return and T is the contract horizon. Alternatively, of course, the investor can annuitize the account value \tilde{W}_T at the then-market rate denoted by \tilde{a}_x . The quantity \tilde{W}_T is obviously unknown in advance and depends on both the selected allocations of the investor and the random performance of underlying market sub-accounts. The subscript x on both annuity factors denotes the age at which the life annuity is priced or issued, for example, age 70 or 75.⁸ Typical market values of \tilde{a}_x under the current interest rate environment might be \$10.2 at the age of $x = 70$, 8.44 at the age of $x = 75$ and 6.72 at the age of $x = 80$. This is the cost of \$1 of annual lifetime income, at the various purchase ages.

One can also think of g_x as the strike price of the GMIB option, although it is not really a put option to sell in the conventional sense, but more of an exchange rate between a guaranteed amount and a lifetime income. Either way the GMIB option pays off, or promises lifetime income in the amount:

$$I = \max \left[\frac{W_0 e^{\eta T}}{g_x}, \frac{\tilde{W}_T(a)}{a_x} \right] \quad (12-1)$$

The justification for this is as follows. If the underlying market and sub-accounts perform poorly (i.e., he earns less than η per annum during the T -year waiting period), the investor is guaranteed the ability to annuitize $W_0 e^{\eta T}$ at the guaranteed annuity factor rate of g_x . On the other hand, if the market value of the account $\tilde{W}_T(a)$ at time T is greater than the guaranteed amount $W_0 e^{\eta T}$, the investor can simply annuitize the (higher)

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account value, at the-then market annuity rates \bar{a}_x . In fact, he does not have to purchase this life annuity from the company that issued the GMIB at all. Instead, he could take his money anywhere and annuitize in the open market.

Note also that $\tilde{W}_T(a)$ is partially under the control of the investor and depends on the asset allocation vector a , which is to be determined. Another way to express this quantity is $\tilde{W}_T(a^* + \varepsilon)$, where a^* was the original allocation in the absence of the GMIB option. Finally, we multiply the guaranteed lifetime income denoted by I , by the then-market annuity factor \bar{a}_x to convert this flow into a lump-sum value at the horizon time T . The objective is to locate an asset allocation vector a^{**} , or incremental allocation ε that maximizes expected utility of wealth:

$$U^{**} = \max_a E[U(I\bar{a}_x)] \quad (12-2)$$

The intuition is as follows. Imagine there is a liquid secondary market for guaranteed lifetime income. In theory, the annuitant could de-annuitize the guaranteed income I and obtain a lump sum in the amount of $I\bar{a}_x$. Thus, the true expected utility of wealth is as displayed in Equation (12-2). Indeed, if the option expires out-of-the-money and the market value of the lifetime income $\tilde{W}_T(a)/\bar{a}_x$ is greater than the guaranteed amount of lifetime income $W_0 e^{\eta T}/g_x$, the mark-to-market value is simply $\tilde{W}_T(a)$ itself. On the other hand, if the option expires in-the-money, the guaranteed lifetime income will kick in and provide income in the amount of $W_0 e^{\eta T}/g_x$ and the market value of this income stream will be: $\bar{a}_x W_0 e^{\eta T}/g_x$. The objective then is to find an asset allocation that maximizes the expected utility of this uniquely defined wealth.

Under the (new) optimal allocation, the expected return from the investment account will be $a^{**}\mu + (1 - a^{**})r - f$, where μ denotes the expected return from the HMR funds, r denotes the LR rate, and the new symbol f denotes the extra fee (a.k.a. mortality and expense charge) for the GMIB. The optimal allocation and the incremental justifiable risk allocations have been generated using a simulation approach, since an analytic approach is impossible and a numerical implementation is equally cumbersome. More specifically, our computational approach conducts simulations for which the GMIB annuity factors g_x are within the vicinity of market annuity factors \bar{a}_x , which can be viewed as a fair GMIB case. We also generated a few in which the GMIB annuity factors were set back relative to market-based annuity rates, so that: $g_x < \bar{a}_x$. For the majority of our simulations, we assumed that the guaranteed return η embedded within the GMIB was 6 percent per annum. Finally, we start with a 55-year-old investor who purchases a GMIB with a 15-year horizon. At the age of 70, he plans with 100 percent certainty to annuitize the account, either under the guarantee

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TABLE 12-4 Optimal Allocations to GMIB Option

<i>% in HMR Without GMIB α^*</i>	<i>Implied Risk Aversion γ</i>	<i>% in HMR with GMIB</i>	<i>CE with GMIB (%) $U^{-1}(U^*)$</i>	<i>CE without GMIB (%) $U^{-1}(U^{**})$</i>
GMIB Annuity Factor $g_{70} = 15$				
30	6.67	100	230	140
40	5.00	100	250	17
50	4.00	100	260	210
GMIB Annuity Factor $g_{70} = 20$				
30	6.67	40–50	200	200
40	5.00	70–100	220	200
50	4.00	100	240	210
GMIB Annuity Factor $g_{70} = 25$				
30	6.67	35–40	200	200
4	5.00	50–60	210	210
50	4.00	70–100	230	220

Source: Authors' computations.

Note: This table illustrates the change in optimal asset allocation when a GMIB is offered on a VA account, as a function of the GMIB annuity factor. For example, an investor with a 40% allocation to risky equity (defined as HMR) reveals a coefficient of relative risk aversion of five. If this individual is offered a GMIB option at the price of \$20 per dollar of lifetime income, he will change his allocation to something between 70% and 100% HMR because of the downside protection. This investor will also experience an increase in certainty equivalent (CE) utility from 200% of initial wealth to 220% of initial wealth. In other words this particular GMIB is welfare enhancing. The underlying parameters were calibrated to fit the historical risk and return parameters of the variable annuity sub-accounts.

(if the HMR asset earns less than 6 percent over the next 15 years) or under the market rates, generating lifetime income of I .

The results for the allocation α^{**} , which maximizes the CRRA utility function relative to the original allocations α^* , appear in Table 12-4. One striking feature is that the 'positivity' of the ε variable—the justification for additional risk exposure—depends on the strike price of the option. The strike price within a GMIB is not immediately obvious and unrelated to the contract's guaranteed investment return (η) was in the vicinity of 6 percent for most of our examples. (Note that sometimes this is expressed with simple compounding, in which case the value must be converted to the true annualized return.) Our point here is more than just that the 'devil is in the details.' Rather, there can be remarkable heterogeneity between various GMIB contract provisions that all appear to offer a 6 percent guaranteed return.

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In the course of extensive simulations to locate the optimal allocation, several interesting results were observed. First, in many instances, the optimal allocations are ‘corner solutions.’ What this means is that when the GMIB annuity factor is favorable to the policyholder, he tends to take on as much risk as allowed by the contract. In a sense, the policyholder has been granted a put option and he maximizes the personal value of this option by taking on as much risk as possible. This is consistent with the empirical evidence presented above (though it assumes full annuitization which we are unable to determine given our data). On the other hand, when the contractual provisions are less favorable, that is, with a higher parameter value of g_x , the optimal allocations in the presence of the GMIB are no longer corner solutions. In some cases, the additional risk exposure levels are only on the order of 5–10 percent.

Conclusion

Portfolio choice and optimal asset allocation over the life cycle are topics that continue to attract both academic and practical interest. This chapter examines how actual allocations to risky assets change when individuals are given ‘downside protection’ in the form of options to annuitize. Specifically, we assess asset allocation inside variable annuity products in which certain insurance riders are available that give investors the option to annuitize at some favorable rate. The data-set we use includes over half a million policyholder accounts, from age 40 to 80, and it permits us to observe their asset allocations, and whether certain riders were selected.

We show that individuals will invest more aggressively when they are granted this type of put option. Indeed, to anyone trained in the valuation and pricing of American-style derivative securities, this notion is straightforward. However, what is less obvious is that these put options are not money-back guarantees but rather they are contingent on annuitization. In other words, the only way to exercise this put is to irreversibly annuitize the contract (at the strike price) in exchange for lifetime income. Thus, if there is some exogenous propensity to avoid annuitization despite its welfare-enhancing properties, it remains to be seen whether these put holders will in fact exercise their options if-and-when they expire in the money.

Our simple model of optimal asset allocation within a GMIB structure only scratches the surface of more accurate representations of the dynamic control problem. Future research should to incorporate the American-style optionality of when to annuitize, as well as the stochasticity of interest rates and perhaps even the credit risk of the insurer, in the event the market collapses. Nevertheless, our result appears robust: more risk is acceptable provided the strike price is sufficiently near the money.

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Notes

¹ This research sits squarely within the portfolio choice literature since we are investigating optimal asset allocations in the presence of various guarantees. We are aware, of course, that in some cases additional risky asset exposure might not be justified.

² The GMIB is closely related to the GMWB, which is not the focus of this analysis. The latter is yet another form of put option contained and selected within many variable annuities. Milevsky and Salisbury (2006) provide a separate analysis of GMWB-based products. Both GMIB and GMWB fall in the category of Guaranteed Living Benefits (GLB). A GLB rider is essentially an insurance rider which provide some sort of portfolio insurance for a VA policyholder, but only once the variable annuity is converted into income.

³ Thus, for example, Goetzmann (1993), Yao and Zhang (2005) as well as Cocco (2005) focus on the role of the housing portfolio; Campbell and Cocco (2003) focus on optimal mortgage choices; Cairns, Blake, and Dowd (2006) examine portfolio choice in defined contribution pension plans, Sundaresan and Zapatero (1997) assess the role of DB pensions, while Dybvig and Liu (2004), and Bodie et al. (2004) model the impact of flexible retirement dates; Jagannathan and Kocherlakota (1996) and Viceira (2001) stress the impact of aging; Faig and Shum (2002) are motivated by the demand for illiquid assets; Koo (1998) as well as Hakansson (1969) and Davis and Willen (2000) model the role of labor income; Dammon, Spatt, and Zhang (2001) focus their attention on capital gains and income taxes; Heaton and Lucas (2000) focus on the role of background risk. Others go back to basics and extend portfolio choice models to include more sophisticated (and realistic) processes for investment returns, such as Chacko and Viceira (2005) or time-varying and mean-reverting risk premiums, such as Kim and Omberg (1996) or Detemple, Garcia, and Rindisbacher (2003).

⁴ See, for instance, Bodie, Merton, and Samuelson (1992); Horneff et al. (2007); Jagannathan and Kocherlakota (1996); Cocco, Gomes, and Maenhout (2005); and Gomes and Michaelides (2005).

⁵ See, for instance, Bodie and Crane (1997), VanDerhei et al. (1999), Agnew, Balduzzi, and Sunden (2003), and Curcuru et al. (2007). Au: Please check the change in year.

⁶ Examples of these would include Yoo (1994), Bertaut and Starr-McCluer (2001), and Faig and Shum (2006). It is important to remind the reader that Ameriks and Zeldes (2004) show that the age to equity share profile is sensitive to the model specification. In fact, any regression explaining portfolio choice with age can include only two of the possible three variables: age, time, and cohort. While Ameriks and

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Zeldes (2004) find a hump-shaped relationship in a regression with age and time effects, they report an increasing allocation to stocks/equity with age in a regression with age and cohort effects. The majority of researchers, however, consider a cohort effect the least significant among the three and make the assumption that it is equal to zero.

⁷ To assess whether regression results overall are different for GMIB selectors versus nonselectors, we combine the two subsamples into one pooled sample and cannot reject the hypothesis that the age differences are statistically significant.

⁸ We refer the interested reader to Milevsky (2006) for a detailed explanation of the actuarial pricing underlying the annuity.

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